1 Introduction

This set of notes is about numerical efforts using the MITgcm software package to study the climate on the time scale of thousands of years. Specifically we are interested in three related problems: what sets the pole equator temperature gradient; what determines the total meridional energy transport and its partition between the atmosphere and the oceans; and what determines the extent of the polar ice caps. These are analyzed using numerical calculations on water covered globes which we call aqua planets with a number of barriers to sea flow. The models are simpler than other large scale calculations but retain the essence of the processes that they simulate. The simulations are able to recover various qualitative features of the climate that exist in the real world, including realistic solutions to the aforementioned problems. Furthermore the models are flexible enough to be used to make predictions about certain features of the climate in the distant past or the near future.

2 Meridional Energy Transport From Observations

The temperature of the Earth decreases poleward. The Earth absorbs more energy at the equator than at the poles, and this energy is circulated to the poles where it is radiated away. This processes is known as meridional energy transport. The heat flux through the atmosphere into space has been observed in detail by satellites, and this data can be used to determine the meridional heat flux. Because the flux is so great it must be driven by bulk transport of cold or warm air and water. There is an overall circulating current in the atmosphere and a corresponding one in the ocean with a net transport of warm air polewards and cool air equatorward [3]. Suppose that at a given latitude the mass flux of these currents are $\Psi_A$ and $\Psi_O$, for the atmosphere and oceans respectively. Then suppose the temperature difference between the currents at each latitude is $B_A$ and $B_O$. The meridional heat transport at a given latitude would then be:

$$H = H_A + H_O = \Psi_A B_A + \Psi_O B_O.$$  \hspace{1cm} (1)
Similarly it is interesting to determine the ratio of the atmospheric and oceanic heat transports:

\[
\frac{H_A}{H_0} = \frac{\Psi_A B_A}{\Psi_O B_O}.
\]

(2)

The partition between atmospheric and oceanic heat transport is known on Earth. These fluxes can be calculated directly by measuring the speed and temperature of currents. Figure 1 is from Trenbeth and Caron [2] and gives the meridional heat transport as a function of latitude. We see that it has a maximum at 35° degrees where it is 6 Petawatts. The total atmospheric heat flux is roughly four times larger than the oceanic flux.

Figure 1: Meridional Heat Transport

The two cases can be analyzed individually, beginning with the oceanic case. The top of the ocean absorbs most of the solar radiation, and as a result is warm. It also has higher salinity due to evaporation. This warm salty layer is driven by the winds. This zone is typically confined to the upper kilometer of the ocean. Underneath it is an abyssal layer which is cold and has lower salinity. The Abyss is vented by convection at the poles, and upwells in the mid and lower latitudes. At its maximum the meridional overturning circulation transports 1.2 Petawatts of heat. The mass flux of this current is on the order of 40 sverdrups.

As is indicated by figure 1 the meridional heat flux is not uniform as a function of latitude. There are also significant variations in the energy contrast and mass flux with latitude. The thermocline is at its most intense in the tropics and the energy contrast is greatest here. In the polar regions the surface temperatures are lower and the ocean is more well mixed.

The meridional circulation of the atmosphere is coupled to that of the ocean. Evaporation of water in the tropics is carried towards the poles. The atmosphere carries moist static energy as a result of its water content. The formula for moist static energy is:

\[
B_A = C_A T + gz + Lq.
\]

(3)
Here $C_A$ is the specific heat of water, $g$ is gravity, $z$ is the altitude, $L$ is the latent heat, and $q$ is the mixing ratio. The moist static energy is the energy that will be released as heat during rainfall. Most of this moisture falls in the midlatitudes. Overall this process accounts for one half of the mass transport of the atmosphere. The differential heating of the atmosphere at the equator and the poles drives this circulation [3].

The atmospheric energy contrast is extremely low in the tropics, so the energy flux must also be low here. On the other hand the energy contrast is quite large in the midlatitudes due to the moist static energy. Figure 2 shows the moisture distribution in the atmosphere as a function of latitude and pressure. The atmosphere is very well mixed in the tropics and this is where most of the convection occurs.

![Figure 2: Moisture Distribution in the Atmosphere and Ocean](image)

The qualitative nature of energy transport in the atmosphere and the oceans is quite different. The ocean convects at the poles while the atmosphere convects at the equator. As a result the energy transport is stronger in the ocean in the equatorial regions. The opposite is true everywhere else. On average the atmospheric mass transport is four times greater than that of the ocean, while the energy contrast is the same. These facts are known from observational studies. If we redefine one Sverdrup as $10^9$ kilograms per second of any substance then the atmosphere is a one hundred twenty Sverdrup flow while the ocean is only a thirty Sverdrup flow. Figure 3 shows this graphically. The energy contrast and mass transport in the atmosphere are calculated using satellite observations of the mass flux between adjacent moist static energy surfaces. Given these observations it is interesting to wonder what properties of the Earth and its climate cause this partition, and to determine how the partition has been different in the past and how it might change in the future.
3  Modeling Hierarchies and MITgcm

The physics of the oceans and the atmosphere are extremely complex. In many cases the most interesting thing is the large scale behavior. Models of the atmosphere and oceans can range from simple to complex. In the first extreme are box models which divide the ocean and atmosphere into a small number of homogeneous boxes. In the other extreme are large scale general circulation models which discretize the Earth and use numerical methods to approximate the evolution of the atmospheres and oceans. The disadvantage of such complex models is the number of free parameters that can be tuned to cause the model to reproduce observations. Instead of being simulations of the physics these can become simulations of the phenomena. This can obscure the true physical causes for a certain phenomenon and thus make the future predictions of the model unreliable.

Given that scientists have adopted the full range of models for global oceanic and atmospheric circulation it is worthwhile to think about the philosophy of modeling in general. This work shuns the overemphasis on complexity and seeks to use models that correspond to actual physics. The idea is to study a given phenomenon by using a hierarchy of models. One begins with the simplest reasonable model and slowly increases its resolution and features. Here our modeling hierarchy will begin with the aqua planet, which is a planet whose surface is entirely covered with water. To simulate the effect of continents, ridges that block ocean circulation are added to the model, but no additional parameters are introduced. After inserting a ridge we can further increase the analogy by putting a gap in the ridge to represent the Drake Passage. The final step in this study is to consider the double drake, which has two ridges with gaps near the south pole. This hierarchy uses a reasonable model of the physics and should produce interesting results because it captures much of the essence of the geography.

The model that will be used to generate experimental results is MITgcm, which is a general circulation model developed at MIT. It sets itself apart from other models because
of its relative simplicity. It is a realistic physical simulation capable of capturing certain physical process[1].

4 Climate of Aqua Planets

We are interested in qualitative studies of the climate and the ocean circulation using large scale GCM models. The starting point for these studies is the so-called aqua planet. It is a landless rotating planet with a five kilometer deep ocean that is dimensionally modeled after the Earth. Using the data from the simulations we will be able to calculate the partition of energy transport between the atmosphere and the oceans and to compare the qualitative features of this transport. Later when more geography is added to the model it will be possible to create a more nuanced understanding of the global energy circulation. The time scale of each simulation is on the order of a couple of thousand years, and this take one or two weeks of computer time.

Figure 4 shows the sea ice present in the model after it has equillibrated. Ice caps form at each pole and are quite large. The ice reaches a maximum depth of seventeen meters at the actual poles. In the absence of topography the oceans are dominated by circumpolar currents at every latitude. The figure 5 shows the strength of the currents in the atmosphere and ocean at each latitude, depth, pressure, and potential temperature. The currents are quite strong at the equator and then reverse direction at the midlatitudes. The strength drops close to the poles. Combined with figure 6, which shows the humidity and salinity we see that the surface of the ocean in the lower latitudes is warm and salty, whereas the abyss is cooler and fresh. This is very similar to Earth. Similarly in the atmosphere the warm air near the tropics is very moist and the cold air near the poles is quite dry.

![Aquaplanet with Ice](image)

Figure 4: Sea Ice and Water Temperatures in the Aqua Planet

The atmospheric circulation has some qualitative features that exist in the Earth's atmosphere. The beta effect is quite important in driving these. For example figure 7a shows the vertical circulation in both the atmosphere and ocean. There are two clearly defined 'Hadley' cells over the tropics.

One remarkable property of this relatively simple model is that the meriodonal energy transport is very similar to that of the Earth. Consider the four figures demonstrating the mass transport and energy transport in aqua planet and on Earth, i.e. figure 3 and figures 7b, c, and d. The mass transport in the two cases are very close and so is the
meridional energy transport. The fact that the shapes of the curves are so similar is astonishing given that the aqua planet has no land. This in many ways represents a victory of the philosophy of modelling hierarchy discussed in the previous section. Although not every feature is captured by this model many of the large scale ones are.
Figure 7: Results from the Aqua Planet: stream function in the ocean and in the atmosphere (a), mass transport (b) and meridional heat transport in Aqua Planet (c) and in Earth (d)
We can use the meridional energy transport data to understand the sea ice caps that are present on the aqua planet. The heat transport drops significantly at the poles. This drop is primarily due to a decline in the mass transport at the poles. In particular the ocean energy transport goes to zero.

5 Role of Geometrical Constraints on Ocean Energy Transport

The coasts are an important constraint for the global circulation in the ocean. The aim of this section is to introduce the impact of the topographic constraints on the flow. Three variations of the aquaplanet model are discussed (figure 8):

- The ridge model: a ridge going from North pole to South pole blocks the zonal flow in the ocean.
- The drake model: similar to the ridge model but with a passageway representing Drake Passage.
- The double drake model: the ocean is shared into a small and a big basin which can communicate by passageways at the south pole.

In these models, the atmosphere is not blocked by a barrier.

![Figure 8: Models with different geometrical constraints: aquaplanet (a), ridge model (b), drake model (c) and double drake model (d).](image)

5.1 Phenomenology

In the ridge model (figure 9a), the oceanic circulation is divided into gyres and a counter current is present along the equator. Moreover, there is no ice cap over the pole. The passageway in the drake model (figure 9b) allows the presence of a circumpolar current. In this case, unlike the ridge model, an ice cap covers the South pole.

Unlike the aquaplanet, there is no subtropical cell driven by the balance between winds and pressure gradients in the ridge model (figure 10b). A deep convection is present at the two poles. In the drake model (figure 10c), convection is active in the warm hemisphere i.e. the northern one where there is no ice. This yields a strong overturning circulation in the northern hemisphere whereas it is tiny in the southern one and then ice can be sustained over the south pole. The salinity is sharply asymmetric in the drake model (figure 11c):
Figure 9: Sea surface temperature computed with the ridge model (a) and the drake model (b).

The salt water is pushed towards the north pole and fresh water towards the south pole. In all models with an oceanic barriers, the atmosphere contains more moisture over the pole than in the aquaplanet model.

Figure 10: Overturning circulation expressed in sverdrup here redefined as $1\text{Sv} = 10^9 \text{ kg. s}^{-1}$ in the aquaplanet model (a), the ridge model (b) and the drake model (c). The red area represents the intensity of the convection.
5.2 Heat Transport and its Partition

The ridge enhances the heat transport in ocean towards the poles (figure 12) explaining the absence of ice. In the drake model, there is less heat transport in the southern hemisphere and consequently an ice cap can be sustained over this pole. In the case of a ridge with an equatorial passageway, the heat transport is enhanced leading to warmer poles (figure 13). Indeed, due to less pressure gradients at the equator, winds are balanced by ageostrophic poleward flow.

In this section, theoretical considerations of a 2-box model will be presented. A box is shared between two parts: the first part represents the equatorial zone and the second one is the polar area (figure 14). A meridional energy flux $f$ is transfered from the equatorial
box to the polar box. Each box undergoes an incoming solar radiation \( s \) and an outgoing longwave radiation \( i \).

\[
\begin{align*}
    a_e s_e - i_e - f & = 0, \\
    a_p s_p - i_p + f & = 0
\end{align*}
\]

with \( a \) the co-albedo.

Each quantity (\( s, i \) or \( a \)) can be decomposed as

\[
\begin{align*}
    (.)_e & = \bar{(.)} + \frac{1}{2} \Delta (.) , \\
    (.)_i & = \bar{(.)} - \frac{1}{2} \Delta (.)
\end{align*}
\]

where \( \bar{(.)} \) is for the global average of \( (.) \) and \( \Delta (.) = (.)_e - (.)_i \).

Using equations 4 and 5, \( f \) can be expressed as

\[
f = \frac{1}{2} \left( \Delta s \bar{a} + \bar{s} \Delta a + \frac{1}{2} \Delta a \Delta s - \Delta i \right).
\]

Stone [4] argued that \( \bar{s} \Delta a \) is balanced by \( \Delta i \) and that \( \frac{1}{2} \Delta a \Delta s \) is negligible (see figure 15). Therefore \( \Delta s \bar{a} \) is the predominant term in the meridional energy flux

\[
f \simeq \frac{1}{2} \Delta s \bar{a}.
\]
It is a reasonable gross estimate but in error by order 20%. This means that the total heat transport only depends on planetary albedo, solar constant and radius of planet. However if ice caps cover the poles, the excess of albedo \( \Delta a \) is important and the \( \Delta s \bar{a} \) effects can not be ignored.

<table>
<thead>
<tr>
<th>Term</th>
<th>( \Delta s \bar{a} )</th>
<th>( \bar{s} \Delta a )</th>
<th>( \frac{1}{2} \Delta s \Delta a )</th>
<th>( -\Delta i )</th>
<th>( f )</th>
<th>Heat Transport</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aqua</td>
<td>0.240</td>
<td>0.086</td>
<td>-0.016</td>
<td>-0.021</td>
<td>0.289</td>
<td>6.25PW</td>
</tr>
<tr>
<td>Ridge</td>
<td>0.249</td>
<td>0.023</td>
<td>-0.004</td>
<td>0.002</td>
<td>0.270</td>
<td>5.88PW</td>
</tr>
<tr>
<td>Drake, NH</td>
<td>0.250</td>
<td>0.024</td>
<td>-0.004</td>
<td>0.007</td>
<td>0.277</td>
<td>6.03PW</td>
</tr>
<tr>
<td>Drake, SH</td>
<td>0.239</td>
<td>0.087</td>
<td>-0.016</td>
<td>-0.026</td>
<td>0.285</td>
<td>6.21PW</td>
</tr>
</tbody>
</table>

Figure 15: Estimates of the different terms of the calculation of the meridional energy flux \( f \) for the different models.

5.3 Double Drake Model

In the double drake model, the ocean is divided into a small and a large basin which can communicate in the southern hemisphere (figure 8d). The small basin is warmer and more salty than the large one (figure 16). The water is reduced in the small basin by evaporation and precipitates in the large basin. The salt water in the small basin conveys and thus a self-sustained overturning circulation is present in the small basin (figure 17). The convection takes place preferentially in the small basin.

Figure 16: Results of the double drake model: sea surface temperature (a), sea surface salinity (b) and sea surface density (c).

The agreement in heat transport by atmosphere and ocean between today’s climate and the double drake model is good despite the straightforward geometrical constraints of the model (figure 18). In particular, the asymmetry in the ocean heat transport between the two hemisphere is well reproduced.
6 Conclusion

Some points deserve to be emphasized:

- Energy flux partition can be rationalized by

\[
\frac{H_A}{H_O} = \frac{\Psi_A}{\Psi_O} \times \frac{\Delta B_A}{\Delta B_O}.
\]

The dominance of $H_A$ over $H_O$ is a consequence of $\Psi_A \gg \Psi_O$. 

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• Although $H_O$ is small, it plays a crucial role e.g. in controlling sea ice extent. The climates of the different models introduced above are very different to one-another especially due to the presence/absence of ice at the poles.

• Oceanic energy transport is primarily achieved by wind-driven currents (not buoyancy-driven flow).

• $f \approx \frac{1}{2} \Delta s \bar{a}$ is a good predictor of total energy transport in warm climate. However $\Delta s \bar{a}$ effects are large in cold climates and can not be ignored.

• Opening of drake passage has a profound effect on global circulation: it is a major source of inter-hemispheric asymmetry.

Some challenges of this work are:

• Multiple equilibria: indeed, stable solutions without ice can be found for the aquaplanet.

• Overlaying biogeochemical cycles in these models.

• Connecting to paleo climate: the different geometrical constraints can be seen as the geological evolution (figure 19).

• Further collaborations: include land.
Figure 19: Geological evolution
References

[1] *MITgcm Online Documentation.*

