Assuming that each plume is passively advected by the entrainment field of the other, the rate of change of the plumes' separation with height is given by the ratio of the vertical and horizontal velocities at the plume axis. As both plumes are deflected equally, the rate is

$$\frac{dx}{dz} = -2\frac{1}{w}\frac{\alpha b w}{x}. \tag{45}$$

Substituting $b = 6\alpha z/5$, $\lambda = z/x_0$, and $\phi = x/x_0$ gives

$$\frac{d\phi}{d\lambda} = -\frac{12\alpha^2}{5}\frac{\lambda}{\phi}, \tag{46}$$

which can be integrated to give

$$\phi^2 - \phi_0^2 = \frac{12\alpha^2}{5}(\lambda_0^2 - \lambda^2), \tag{47}$$

where $\phi_0 = 1$ and $\lambda_0 = 0$. Using (38) to (47) we obtain

$$\lambda_m = \frac{1}{\alpha}\sqrt{\frac{25}{132}} \approx \frac{0.44}{\alpha}. \tag{48}$$

In other word, the merging height is predicted as

$$z_m \approx \frac{0.44}{\alpha}x_0. \tag{49}$$

Note that the use of (38) derived for the non-interacting model is appropriate here as the correction for drawing together of the plumes assumes that they are passively advected only. The radial growth rate of each plume is assumed to be unaffected by this process.

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